RUHR-UNIVERSITÄT BOCHUM

A formal Security Analysis of the EdDSA Signature Scheme

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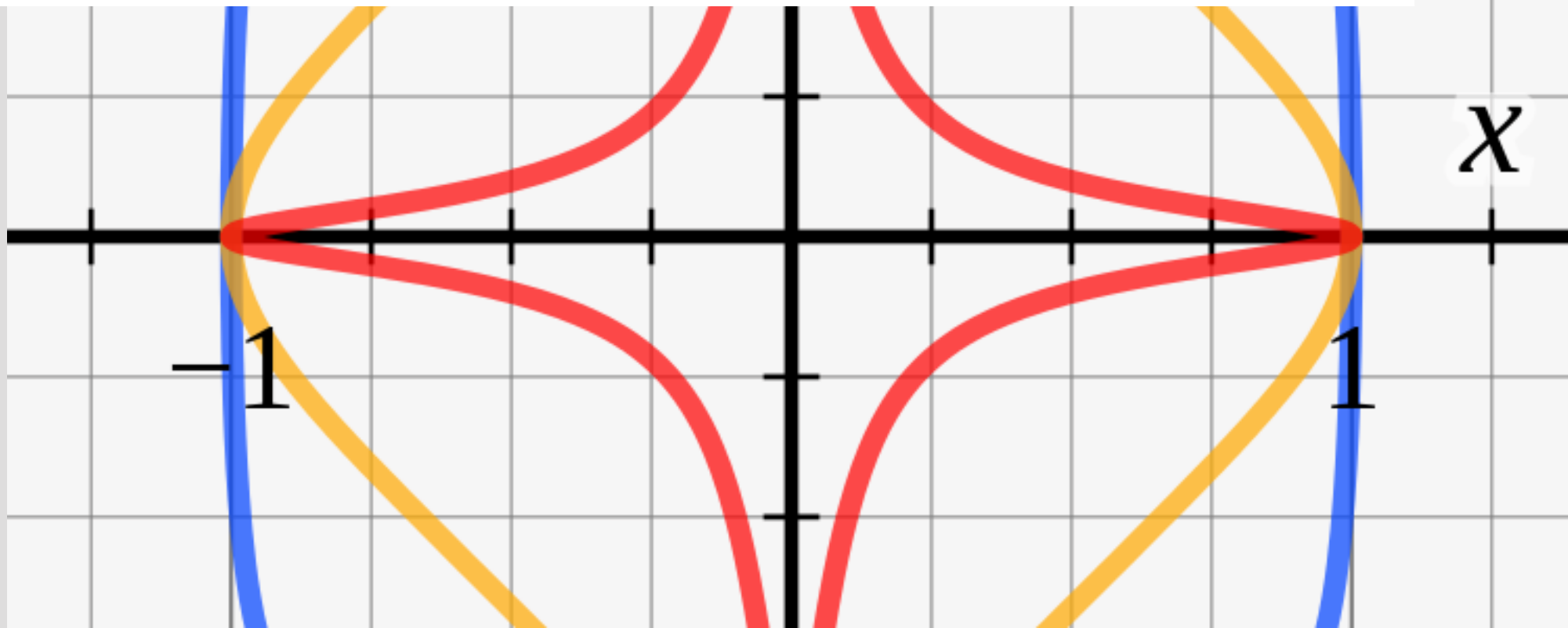

Outline

Overview

What is this thesis about?

1. Providing a tight security proof for the EdDSA signature scheme
2. Showing the security level of concrete instantiations of EdDSA

Overview

Results

Results of this thesis:

1. EdDSA is tightly secure under Ed-DLog assumption in the single-user setting
2. EdDSA is tightly secure under the Ed-N-DLog-Reveal assumption in the multi-user setting
3. Ed25519 provides 125/124 bits of security in the single/multi-user setting
4. Ed448 provides 221/220 bits of security in the single/multi-user setting

Overview

Results

Results of this thesis:

1. EdDSA is tightly secure under Ed-DLog assumption in the single-user setting
2. EdDSA is tightly secure under the Ed-N-DLog-Reveal assumption in the multi-user setting
3. Ed25519 provides 125/124 bits of security in the single/multi-user setting
4. Ed448 provides 221/220 bits of security in the single/multi-user setting

Motivation

EdDSA is everywhere...

Motivation

No existing tight security proof since publication in 2011

Related Work

- Brendel et al. 2021 [1]: First security proof for Ed25519
- Chalkias et al. 2020 [2]: Analysis of different EdDSA implementations
- Fuchsbauer et al. 2020 [3]: Tight security proof for Schnorr Signatures using AGM

Security Notions

Digital Signature Scheme

RUB

Definition

A digital signature scheme SIG = (KeyGen,Sign,Verify) is a tuple of algorithms.

- **KeyGen**: The key generation algorithm, which upon receiving the security parameter as input outputs a matching tuple of public and private key.
- **Sign**: The signature algorithm, which upon receiving a secret key and a message, outputs a signature for that message.
- **Verify**: The verification algorithm, which upon receiving a public key, a message and a signature, outputs 1 if the signature gets accepted and 0 otherwise.

For the digital signature scheme to be correct, it is required that
$\forall (pk, sk) \in \text{KeyGen}(par), m \in \mathcal{M}, \sigma \in \text{Sign}(sk, m) : \text{Verify}(pk, m, \sigma) = 1$

Security Notions

N-MU-EUF-CMA

Game N-MU-EUF-CMA

for $i \in \{1, 2, ..., N\}$

$(pk_i, sk_i) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$(m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathsf{Sign}(\cdot,\cdot)}(pk_1, pk_2, ..., pk_n)$

return $\exists i \in \{1, 2, ..., N\} : \mathsf{Verify}(pk_i, m^*, \sigma^*) \stackrel{?}{=} 1 \wedge (pk_i, m^*) \notin M$

Oracle *Sign* $(i \in \{1, 2, ..., N\},\ m \in \mathcal{M})$

$\sigma \leftarrow \mathsf{Sign}(sk_i, m)$

$M := M \cup \{(pk_i, m)\}$

return σ

Abb. N-MU-EUF-CMA Security Game

Security Notions

N-MU-SUF-CMA

Game N-MU-SUF-CMA

for $i \in \{1, 2, ..., N\}$

$(pk_i, sk_i) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$(m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathsf{Sign}(\cdot,\cdot)}(pk_1, pk_2, ..., pk_n)$

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Oracle *Sign* $(i \in \{1, 2, ..., N\},\ m \in \mathcal{M})$

$\sigma \leftarrow \mathsf{Sign}(sk_i, m)$

$M := M \cup \{(pk_i, m, \sigma)\}$

return σ

Abb. N-MU-SUF-CMA Security Game

Security Notions

N-MU-EUF-NMA

Game N-MU-EUF-NMA

for $i \in \{1, 2, ..., N\}$

$(pk_i, sk_i) \leftarrow \text{KeyGen}(1^\lambda)$

$(m^*, \sigma^*) \leftarrow \mathcal{A}(pk_1, pk_2, pk_n)$

return $\exists i \in \{1, 2, ..., N\} : \text{Verify}(pk_i, m^*, \sigma^*) \stackrel{?}{=} 1$

Abb. N-MU-EUF-NMA Security Game

Random Oracle Model (ROM)

- Hash functions are modeled as public oracle
- Oracle behaves like a true random function
- Challenger can observe all inputs
- Challenger can program the random oracle

Algebraic Group Model (AGM)

RUB

- Adversary has to provide a representation of all group elements

Generic Group Model (GGM)

- GGM hides all group-specific representation of group elements
- Adversary works with random labels instead of actual group elements

Standards

- Paper by Bernstein et al. [4, 5]
- RFC 8032 [6]
- FIPS 186-5 [7]

The EdDSA Signature Scheme

Parameter	Description
q	An odd prime power q. EdDSA uses an elliptic curve over the finite field $\mathbb{F}_q$.
b	An integer b with $2^{b-1} > q$. The bit size of encoded points on the twisted Edwards curve.
$Enc(\cdot)$	A $(b-1)$-bit encoding of elements in the underlying finite field.
$H(\cdot)$	A cryptographic hash function producing $2b$-bit output.
c	The cofactor of the twisted Edwards curve.
n	The number of bits used for the secret scalar of the public key.
a, d	The curve parameter of the twisted Edwards curve.
B	A generator point of the prime order subgroup of E.
L	The order of the prime order subgroup.
$H'(\cdot)$	A prehash function applied to the message prior to applying the Sign or Verify procedure.

Tab. Parameter of the EdDSA signature scheme

The EdDSA Signature Scheme

Parameter	Description
b	An integer b with $2^{b-1} > q$. The bit size of encoded points on the twisted Edwards curve.
$H(\cdot)$	A cryptographic hash function producing $2b$-bit output.
c	The cofactor of the twisted Edwards curve.
n	The number of bits used for the secret scalar of the public key.
B	A generator point of the prime order subgroup of E.
L	The order of the prime order subgroup.

Tab. Parameter of the EdDSA signature scheme

The EdDSA Signature Scheme

KeyGen

$k \leftarrow \{0,1\}^b$

$(h_0, h_1, ..., h_{2b-1}) := H(k)$

$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$

$A := sB$

return $(\underline{A}, k)$

The EdDSA Signature Scheme

RUB

$\underline{\textbf{Sign}(k,\ m)}$
$(h_0, h_1, ..., h_{2b-1}) := H(k)$
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$(r'_0, r'_1, ..., r'_{2b-1}) := H(h_b|...|h_{2b-1}|m)$
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$\textbf{ch} := H(\underline{R}|\underline{A}|m)$
$S := (r + \textbf{ch} \cdot s) \pmod{L}$
return $\sigma := (\underline{R}, S)$

The EdDSA Signature Scheme

Sign(k, m)

$(h_0, h_1, ..., h_{2b-1}) := H(k)$ ▷ *Recover secret scalar*
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$(r'_0, r'_1, ..., r'_{2b-1}) := H(h_b|...|h_{2b-1}|m)$
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$\mathbf{ch} := H(\underline{R}|\underline{A}|m)$
$S := (r + \mathbf{ch} \cdot s) \pmod{L}$
return $\sigma := (\underline{R}, S)$

The EdDSA Signature Scheme

RUB

$\underline{\textbf{Sign}(k,\ m)}$
$(h_0, h_1, ..., h_{2b-1}) := H(k)$
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$(r'_0, r'_1, ..., r'_{2b-1}) := H(h_b|...|h_{2b-1}|m)$ ▷ *Calculate commitment*
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$\textbf{ch} := H(\underline{R}|\underline{A}|m)$
$S := (r + \textbf{ch} \cdot s) \pmod{L}$
return $\sigma := (\underline{R}, S)$

The EdDSA Signature Scheme

RUB

$\underline{\textbf{Sign}(k,\ m)}$
$(h_0, h_1, ..., h_{2b-1}) := H(k)$
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$(r'_0, r'_1, ..., r'_{2b-1}) := H(h_b|...|h_{2b-1}|m)$
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$\mathbf{ch} := H(\underline{R}|\underline{A}|m)$ ▷ *Calculate challenge*
$S := (r + \mathbf{ch} \cdot s) \pmod{L}$
return $\sigma := (\underline{R}, S)$

The EdDSA Signature Scheme

RUB

$\underline{\textbf{Sign}(k,\ m)}$
$(h_0, h_1, ..., h_{2b-1}) := H(k)$
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$(r'_0, r'_1, ..., r'_{2b-1}) := H(h_b|...|h_{2b-1}|m)$
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$\textbf{ch} := H(\underline{R}|\underline{A}|m)$
$S := (r + \textbf{ch} \cdot s) \pmod{L}$ ▷ *Calculate response*
return $\sigma := (\underline{R}, S)$

The EdDSA Signature Scheme

RUB

$\underline{\textbf{Verify}(\underline{A}, \sigma := (\underline{R}, S), m)}$

return $2^c SB \stackrel{?}{=} 2^c R + 2^c H(\underline{R}|\underline{A}|m)A$

The EdDSA Signature Scheme

Signature Parsing

- Strict parsing: Reject all bitstring representations of $S > L$
- Lax parsing: Allow all bitstring representations of S and work with $S \pmod{L}$

The EdDSA Signature Scheme

Encoding of Group Elements

- Decoding function ensures that point is on curve
- Multiple bitstrings might map to the same point on the curve

The EdDSA' Signature Scheme

KeyGen

$(h_0, h_1, ..., h_{2b-1}) \leftarrow \{0,1\}^{2b}$
$s \leftarrow 2^n + \sum_{i=c}^{n-1} 2^i h_i$
$A := sB$
return $(\underline{A},\ k := (s, h_b|...|h_{2b-1}))$

Sign$(k := (s, h_b|...|h_{2b-1}),\ m)$

$(r'_0, r'_1, ..., r'_{2b-1}) := RF(h_b|...|h_{2b-1}|m)$
$r := \sum_{i=0}^{2b-1} 2^i r'_i$
$R := rB$
$S := (r + sH(\underline{R}|\underline{A}|m)) \pmod{L}$
return $\sigma := (\underline{R}, S)$

Abb. Generic description of the algorithms KeyGen, Sign and Verify used by the EdDSA' signature scheme

The EdDSA' Signature Scheme

RUB

Theorem

Let $\mathcal{A}$ be an adversary against SUF-CMA security of the EdDSA signature scheme. Then

$$Adv^{\text{SUF-CMA}}_{\text{EdDSA}',\mathcal{A}}(\lambda) \leq Adv^{\text{SUF-CMA}}_{\text{EdDSA},\mathcal{A}}(\lambda) + \frac{2(q_h + 1)}{2^b}.$$

Single-User Security

Theorem (Security of EdDSA with strict parsing in the single-user setting)

Let $\mathcal{A}$ be an adversary against the SUF-CMA security of EdDSA with strict parsing, making at most q_h hash queries and q_o oracle queries, and $\mathbb{G}$ be a group of prime order L. Then,

$$Adv^{SUF\text{-}CMA}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{Ed\text{-}DLog}_{E,n,c,L,\mathcal{B}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}$$

Single-User Security

Theorem (Security of EdDSA with lax parsing in the single-user setting)

Let $\mathcal{A}$ be an adversary against the EUF-CMA security of EdDSA with lax parsing, making at most q_h hash queries and q_o oracle queries, and $\mathbb{G}$ be a group of prime order L. Then,

$$Adv^{EUF\text{-}CMA}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{Ed\text{-}DLog}_{E,n,c,L,\mathcal{B}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}$$

Single-User Security

$$\text{Ed-DLog} \overset{\text{AGM}}{\Rightarrow} \text{Ed-IDLOG} \overset{\text{ROM}}{\Rightarrow} \text{EUF-NMA} \overset{\text{ROM}}{\Rightarrow} \text{SUF-CMA}_{\text{EdDSA sp}}$$

$$\text{Ed-DLog} \overset{\text{AGM}}{\Rightarrow} \text{Ed-IDLOG} \overset{\text{ROM}}{\Rightarrow} \text{EUF-NMA} \overset{\text{ROM}}{\Rightarrow} \text{EUF-CMA}_{\text{EdDSA lp}}$$

Single-User Security

EUF-NMA $\overset{\text{ROM}}{\Rightarrow}$ SUF-CMA$_{\text{EdDSA sp}}$/EUF-CMA$_{\text{EdDSA lp}}$

Theorem ([1])

Let $\mathcal{A}$ be an adversary against SUF-CMA, making at most q_h hash queries and q_o oracle queries, and let $\mathbb{G}$ be a group of prime order L. Then,

$$Adv^{\text{SUF-CMA}}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{\text{EUF-NMA}}_{\mathbb{G},\mathcal{B}}(\lambda) + \frac{q_o q_h}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}.$$

Single-User Security

EUF-NMA $\overset{\text{ROM}}{\Rightarrow}$ SUF-CMA$_{\text{EdDSA sp}}$

Proof Idea:

- Simulate signatures without private key
  1. Choose commitment and solution uniformly at random
  2. Calculate corresponding challenge
  3. Program random oracle to output that challenge for the signature
- Forward random oracle quries to challenger
- A valid signature forgery in the SUF-CMA game is also a valid forgery in the UF-NMA game

Single-User Security

Ed-IDLOG $\overset{\text{ROM}}{\Rightarrow}$ EUF-NMA

Game Ed-IDLOG

$a \leftarrow \{2^{n-1}, 2^{n-1}+2^c, ..., 2^n - 2^c\}$

$A := aB$

$s^* \leftarrow \mathcal{A}^{Chall(\cdot)}(A)$

return $\exists (R^*, \mathbf{ch}^*) \in \mathrm{Q} : R^* = 2^c s^* B - 2^c \mathbf{ch}^* A$

Oracle *Chall* $(R_i \in \mathbb{G})$

$\mathbf{ch}_i \leftarrow \{0,1\}^{2b}$

$\mathrm{Q} := \mathrm{Q} \cup \{(R_i, \mathbf{ch}_i)\}$

return $\mathbf{ch}_i$

Abb. Ed-IDLOG

Single-User Security

Ed-IDLOG $\overset{\text{ROM}}{\Rightarrow}$ EUF-NMA

Theorem

Let $\mathcal{A}$ be an adversary against EUF-NMA. Then,

$$Adv^{EUF\text{-}NMA}_{\mathbb{G},\mathcal{A}}(\lambda) = Adv^{Ed\text{-}IDLOG}_{\mathbb{G},\mathcal{B}}(\lambda).$$

Single-User Security

Ed-IDLOG $\overset{\text{ROM}}{\Rightarrow}$ EUF-NMA

Proof Idea:

- Forward random oracle quries to *Chall* oracle
- A valid signature forgery provides a valid solution for Ed-IDLOG

Single-User Security

Ed-DLog $\overset{\text{AGM}}{\Rightarrow}$ Ed-IDLOG

Game Ed-DLog

$a \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, ..., 2^n - 2^c\}$

$A := aB$

$a' \leftarrow \mathcal{A}(A)$

return $a \overset{?}{=} a'$

Abb. Ed-DLog

Single-User Security

Ed-DLog $\overset{\text{AGM}}{\Rightarrow}$ Ed-IDLOG

Theorem

Let $\mathcal{A}$ be an adversary against Ed-IDLOG with $\mathbb{G}$ being a cyclic group of prime order L, making at most q_o oracle queries. Then

$$Adv^{\text{Ed-IDLOG}}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{\text{Ed-DLog}}_{\mathbb{G},\mathcal{B}}(\lambda) + \frac{q_o}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}.$$

Single-User Security

Ed-DLog $\overset{\text{AGM}}{\Rightarrow}$ Ed-IDLOG

Proof Idea:

- Adversary provides a valid solution: $R^* = 2^c s^* B - 2^c \mathbf{ch}^* A$
- Adversary also provides: $R^* = r_1 B + r_2 A$
- Rewrite equations: $A = (2^c s^* - r_1)(r_2 + 2^c \mathbf{ch}^*)^{-1} B$

Multi-User Security

Theorem (Security of EdDSA with strict parsing in the multi-user setting)

Let $\mathcal{A}$ be an adversary against the N-MU-SUF-CMA security of EdDSA with strict parsing, receiving N public keys and making at most q_h hash queries and q_o oracle queries, and $\mathbb{G}$ be a group of prime order L. Then,

$$Adv^{MU\text{-}SUF\text{-}CMA}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{Ed\text{-}N\text{-}DLog\text{-}Reveal}_{E,n,c,L,\mathcal{B}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o N}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}$$

Multi-User Security

Theorem (Security of EdDSA with lax parsing in the multi-user setting)

Let $\mathcal{A}$ be an adversary against the N-MU-EUF-CMA security of EdDSA with lax parsing, receiving N public keys and making at most q_h hash queries and q_o oracle queries, and $\mathbb{G}$ be a group of prime order L. Then,

$$Adv^{MU\text{-}EUF\text{-}CMA}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{Ed\text{-}N\text{-}DLog\text{-}Reveal}_{E,n,c,L,\mathcal{B}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o N}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}$$

Multi-User Security

- MU-EUF-NMA $\overset{\text{ROM}}{\Rightarrow}$ MU-SUF-CMA$_{\text{EdDSA sp}}$
- MU-EUF-NMA $\overset{\text{ROM}}{\Rightarrow}$ MU-EUF-CMA$_{\text{EdDSA lp}}$
- Ed-N-DLog-Reveal $\overset{\text{AGM}}{\Rightarrow}$ N-Ed-IDLOG $\overset{\text{ROM}}{\Rightarrow}$ MU-EUF-NMA

Multi-User Security

Ed-N-DLog-Reveal $\overset{\text{AGM}}{\Rightarrow}$ N-Ed-IDLOG

Theorem

Let $\mathcal{A}$ be an adversary against N-Ed-IDLOG with $\mathbb{G}$ being a cyclic group of prime order L, receiving N public keys and making at most q_o oracle queries. Then

$$Adv^{N\text{-}Ed\text{-}IDLOG}_{\mathbb{G},\mathcal{A}}(\lambda) \leq Adv^{Ed\text{-}N\text{-}DLog\text{-}Reveal}_{\mathbb{G},\mathcal{B}}(\lambda) + \frac{q_o N}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}.$$

Multi-User Security

Ed-N-DLog-Reveal $\overset{\text{AGM}}{\Rightarrow}$ N-Ed-IDLOG

Game N-Ed-IDLOG

for $i \in \{1, 2, ..., N\}$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, ..., 2^n - 2^c\}$

$A_i := a_i B$

$s^* \leftarrow \mathcal{A}^{Chall(\cdot)}(A_1, A_2, ..., A_N)$

return $\exists (R^*, \mathbf{ch}^*) \in \mathrm{Q}, i \in \{1, 2, ..., N\} \in : R^* = 2^c s^* B - 2^c \mathbf{ch}^* A_i$

Oracle $Chall$ $(R_i \in \mathbb{G})$

$\mathbf{ch}_i \leftarrow \{0,1\}^{2b}$

$\mathrm{Q} := \mathrm{Q} \cup \{(R_i, \mathbf{ch}_i)\}$

return $\mathbf{ch}_i$

Abb. N-Ed-IDLOG

Multi-User Security

Ed-N-DLog-Reveal $\overset{\text{AGM}}{\Rightarrow}$ N-Ed-IDLOG

Game Ed-N-DLog-Reveal

for $i \in \{1, 2, ..., N\}$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, ..., 2^n - 2^c\}$

$A_i := a_i B$

$(a'_1, a'_2, ..., a'_N) \leftarrow \mathcal{A}^{DL(\cdot)}(A_1, A_2, ..., A_N)$

return $(a_1, a_2, ..., a_N) \overset{?}{=} (a'_1, a'_2, ..., a'_N)$

Oracle $DL(j \in \{1, 2, ..., N\})$ ▷ *max. one query*

return $\{a_i | i \in \{1, 2, ..., N\} \backslash \{j\}\}$

Abb. Ed-N-DLog-Reveal

Multi-User Security

Ed-N-DLog-Reveal $\overset{\text{AGM}}{\Rightarrow}$ N-Ed-IDLOG

Proof Idea:

- Similar to single-user proof
- Query discrete logarithms of all but one challenge group element A_i
- Construct a representation $R^* = r_1 B + r_2 A_i$
- Calculate discrete logarithm of A_i

Ed-DLog

RUB

Theorem

Let n and c be positive integers. Consider a twisted Edwards curve E with a cofactor of 2^c and a generating set consisting of $(B, E_2, ..., E_m)$. Among these, let B be the generator of the largest prime order subgroup with an order of L. Let $\mathcal{A}$ be a generic adversary making at most q_g group operations. Then,

$$Adv^{\text{Ed-DLog}}_{E,n,c,L,\mathcal{A}} \leq \frac{(q_g + 3)^2 + 1}{2^{n-1-c}}.$$

Ed-N-DLog-Reveal

RUB

Theorem

Let n, N, c be positive integers. Consider a twisted Edwards curve E with a cofactor of 2^c and a generating set consisting of $(B, E_2, ..., E_m)$. Among these, let B be the generator of the largest prime order subgroup with an order of L. Let $\mathcal{A}$ be a generic adversary against Ed-N-DLog-Reveal receiving N group elements as challenge and making at most q_g group operations queries. Then,

$$Adv^{\textit{Ed-N-DLog-Reveal}}_{E,n,c,L,\mathcal{A}} \leq \frac{2(q_g + N + 2)^2 + 1}{2^{n-1-c}}.$$

Ed-N-DLog-Reveal

RUB

Lemma (Schwartz-Zippel lemma [8])

Let L be a prime number and $P \in \mathbb{F}_L[X_1, ..., X_n]$ be a non-zero polynomial of total degree $d \geq 0$ over a field $\mathbb{F}_L$. Let S be a finite subset of $\mathbb{F}_L$ and let x be selected uniformly at random from S. Then

$$\Pr[P(x) = 0] \leq \frac{d}{|S|}.$$

Ed-N-DLog-Reveal

Game G_0 / G_1 / G_2 / G_3 / G_4
for $i \in \{1, 2, \ldots, N\}$
  $a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$
  $A_i \coloneqq a_i B$
$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$
return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle DL($j \in \{1, 2, \ldots, N\}$)
return $\{a_i | i \in \{1, 2, \ldots, N\} \setminus \{j\}\}$

Oracle GOp($x, y \in \mathsf{S}, b \in \{0, 1\}$)
  return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure Enc($X \in E$)
If $\sum[X] = \perp$ **then**
  $\sum[X] \leftarrow \{0,1\}^{\lceil log_2(|E|) \rceil} \setminus \mathsf{S}$
  $\mathsf{S} \coloneqq \mathsf{S} \cup \{\sum[X]\}$
return $\sum[X]$

Abb. G_0

Ed-N-DLog-Reveal

Game G_0 / G_1 / G_2 / G_3 / G_4

for $i \in \{1, 2, \ldots, N\}$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$

$A_i := (a_i, 0, \ldots, 0)$

$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$

return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle $\mathrm{DL}(j \in \{1, 2, \ldots, N\})$

return $\{a_i | i \in \{1, 2, \ldots, N\} \setminus \{j\}\}$

Oracle $\mathrm{GOp}(x, y \in \mathbf{S}, b \in \{0, 1\})$

return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure $\mathrm{Enc}(X \in \mathbb{Z}_L \times \mathbb{Z}_{ord(E_2)} \times \ldots \times \mathbb{Z}_{ord(E_n)})$

If $\sum[X] = \perp$ **then**

$\sum[X] \leftarrow \{0, 1\}^{\lceil log_2(|E|) \rceil} \setminus \mathbf{S}$

$\mathbf{S} := \mathbf{S} \cup \{\sum[X]\}$

return $\sum[X]$

Abb. G_1

Ed-N-DLog-Reveal

Game G_0 / G_1 / G_2 / G_3 / G_4

for $i \in \{1, 2, \ldots, N\}$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$

$P_i := Z_i$

$A_i := (P_i, 0, \ldots, 0)$

$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$

return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle DL($j \in \{1, 2, \ldots, N\}$)

return $\{a_i | i \in \{1, 2, \ldots, N\} \setminus \{j\}\}$

Oracle GOp($x, y \in \mathbf{S}, b \in \{0, 1\}$)

return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure Enc($X \in \mathbb{Z}_L[Z_1, \ldots, Z_N] \times \mathbb{Z}_{ord(E_2)} \times \ldots \times \mathbb{Z}_{ord(E_n)}$)

Let $X = (P, x_2, \ldots, x_n)$

$\mathbf{P} = \mathbf{P} \cup \{P\}$

$X := (P(\overrightarrow{a}), x_2, \ldots, x_n)$

If $\sum[X] = \perp$ **then**

$\sum[X] \leftarrow \{0, 1\}^{\lceil log_2(|E|) \rceil} \setminus \mathbf{S}$

$\mathbf{S} := \mathbf{S} \cup \{\sum[X]\}$

return $\sum[X]$

Abb. G_2

Ed-N-DLog-Reveal

Oracle DL($j \in \{1, 2, ..., N\}$)

for $P_i \in \mathrm{P}$ ▷ G_3
  Let $P_i = R_i + S_i, R_i \in \mathbb{Z}_L[Z_1, ..., Z_{j-1}, Z_{j+1}, ..., Z_N], S_i \in \mathbb{Z}_L[Z_j]$
  $\mathrm{R} := \mathrm{R} \cup \{R_i\}$
if $\exists R_i, R_j \in \mathrm{R} : R_i(\vec{a}) = R_j(\vec{a}) \wedge R_i \neq R_j$
  $bad_1 := true$
  abort ▷ G_4
for $P_i \in \mathrm{P}$
  $\sum[R_i(\vec{a}) + S_i] = \sum[P_i]$
  $P_i := R_i(\vec{a}) + S_i$
return $\{a_i | i \in \{1, 2, ..., N\} \backslash \{j\}\}$

Abb. $G_3 - G_4$

Ed-N-DLog-Reveal

Game G_4 / G_5 / G_6 / G_7 / G_8

for $i \in \{1, 2, \ldots, N\}$
  $a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$
  $P_i \coloneqq Z_i$
  $A_i \coloneqq (P_i, 0, \ldots, 0)$
$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$

if $\exists P_i, P_j \in \mathbf{P} : P_i(\vec{a}) = P_j(\vec{a}) \wedge P_i \neq P_j$ ▷ G_5
  $bad_2 \coloneqq true$
abort ▷ G_6

return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle DL$(j \in \{1, 2, \ldots, N\})$
…

Oracle GOp$(x, y \in \mathbf{S}, b \in \{0, 1\})$
  return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure Enc$(X \in \mathbb{Z}_L[Z_1, \ldots, Z_N] \times \mathbb{Z}_{ord(E_2)} \times \ldots \times \mathbb{Z}_{ord(E_n)})$
Let $X = (P, x_2, \ldots, x_n)$
$\mathbf{P} = \mathbf{P} \cup \{P\}$
$X \coloneqq (P(\vec{a}), x_2, \ldots, x_n)$
If $\sum[X] = \perp$ **then**
  $\sum[X] \leftarrow \{0, 1\}^{\lceil log_2(|E|) \rceil} \setminus \mathbf{S}$
  $\mathbf{S} \coloneqq \mathbf{S} \cup \{\sum[X]\}$
return $\sum[X]$

Ahh $G_5 = G_6$

Ed-N-DLog-Reveal

Game G_4 / G_5 / G_6 / G_7 / G_8

for $i \in \{1, 2, \ldots, N\}$
  $a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$
  $P_i \coloneqq Z_i$
  $A_i \coloneqq (P_i, 0, \ldots, 0)$
$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$
if $\exists P_i, P_j \in \mathsf{P} : P_i(\vec{a}) = P_j(\vec{a}) \wedge P_i \neq P_j$
  $bad_2 \coloneqq true$
  abort
return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle DL($j \in \{1, 2, \ldots, N\}$)
…

Oracle GOp($x, y \in \mathsf{S}, b \in \{0, 1\}$)
  return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure Enc($X \in \mathbb{Z}_L[Z_1, \ldots, Z_N] \times \mathbb{Z}_{ord(E_2)} \times \ldots \times \mathbb{Z}_{ord(E_n)}$)
Let $X = (P, x_2, \ldots, x_n)$
$\mathsf{P} = \mathsf{P} \cup \{P\}$
~~$X \coloneqq (P(\vec{a}), x_2, \ldots, x_n)$~~
If $\sum[X] = \perp$ **then**
  $\sum[X] \leftarrow \{0,1\}^{\lceil log_2(|E|)\rceil} \setminus \mathsf{S}$
  $\mathsf{S} \coloneqq \mathsf{S} \cup \{\sum[X]\}$
return $\sum[X]$

Abb. G_7

Ed-N-DLog-Reveal

Game G_4 / G_5 / G_6 / G_7 / G_8

for $i \in \{1, 2, \ldots, N\}$

~~$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$~~

$P_i := Z_i$

$A_i := (P_i, 0, \ldots, 0)$

$(a'_1, a'_2, \ldots, a'_N) \leftarrow \mathcal{A}^{GOp(\cdot,\cdot,\cdot), DL(\cdot)}(Enc(B), Enc(E_2), \ldots, Enc(E_m), Enc(A_1), \ldots, Enc(A_N))$

for $i \in \{1, 2, \ldots, N\}$

if $a_i = \perp$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$

if $\exists P_i, P_j \in \mathsf{P} : P_i(\overrightarrow{a}) = P_j(\overrightarrow{a}) \wedge P_i \neq P_j$

$bad_2 := true$

abort

return $(a_1, a_2, \ldots, a_N) \stackrel{?}{=} (a'_1, a'_2, \ldots, a'_N)$

Oracle DL($j \in \{1, 2, \ldots, N\}$)

for $i \in \{1, 2, \ldots, N\} \setminus \{j\}$

$a_i \leftarrow \{2^{n-1}, 2^{n-1} + 2^c, \ldots, 2^n - 2^c\}$

…

Oracle GOp($x, y \in \mathsf{S}, b \in \{0, 1\}$)

return $Enc(\sum^{-1}[x] + (-1)^b \sum^{-1}[y])$

Procedure Enc($X \in \mathbb{Z}_L[Z_1, \ldots, Z_N] \times \mathbb{Z}_{ord(E_2)} \times \ldots \times \mathbb{Z}_{ord(E_n)}$)

…

Abb. G_8

Concrete Security

RUB

Definition (Success Ratio [9])

Let adversary $\mathcal{A}$ be an adversary with runtime $\text{Time}(\mathcal{A})$ and advantage $\text{Adv}_{\mathcal{A}}$. Its success ratio is defined as following:

$$SR(\mathcal{A}) = \frac{\text{Adv}_{\mathcal{A}}}{Time(\mathcal{A})}.$$

Definition (Bit Security [9])

A cryptographic scheme has κ bit security if the success ratio of all adversaries with a runtime $Time(\mathcal{A}) \leq 2^{\kappa}$ is upper bounded by $2^{-\kappa}$.

Concrete Security

Ed25519

$$
\begin{aligned}
SR(\mathcal{A}) &\leq \frac{\mathsf{Adv}^{\text{SUF-CMA}}_{\mathbb{G},\mathcal{A}}(\lambda)}{Time(\mathcal{A})} \\
&\leq \frac{\mathsf{Adv}^{\text{Ed-DLog}}_{E,n,c,L,\mathcal{B}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}}{Time(\mathcal{A})} \\
&\leq \frac{\frac{(q_g+3)^2+1}{2^{n-1-c}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}}{Time(\mathcal{A})} \\
&\leq \frac{(2^{125}+3)^2+1}{2^{250}2^{125}} + \frac{2(2^{125}+1)}{2^{256}2^{125}} + \frac{2^{64}2^{125}+2^{64}}{2^{252}2^{125}} \\
&\approx 2^{-125} + 2^{-316} + 2^{-189} \\
&\approx 2^{-125}
\end{aligned}
$$

Concrete Security

Ed25519

$$
\begin{aligned}
SR(\mathcal{A}) &\leq \frac{\mathrm{Adv}^{\text{MU-SUF-CMA}}_{\mathbb{G},\mathcal{A}}(\lambda)}{Time(\mathcal{A})} \\
&\leq \frac{\mathrm{Adv}^{\text{Ed-}N\text{-DLog-Reveal}}_{E,n,c,L,\mathcal{A}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o N}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}}{Time(\mathcal{A})} \\
&\leq \frac{\frac{2(q_g+N+2)^2+1}{2^{n-1-c}} + \frac{2(q_h+1)}{2^b} + \frac{q_o q_h + q_o N}{2^{-\log_2(\lceil \frac{2^{2b}-1}{L} \rceil 2^{-2b})}}}{Time(\mathcal{A})} \\
&\leq \frac{2(2^{125}+2^{35}+2)^2+1}{2^{250}2^{125}} + \frac{2(2^{125}+1)}{2^{256}2^{125}} + \frac{2^{64}2^{125}+2^{64}2^{35}}{2^{252}2^{125}} \\
&\approx 2^{-124} + 2^{-316} + 2^{-189} \\
&\approx 2^{-124}
\end{aligned}
$$

Thank you!

Questions?

J. Brendel, C. Cremers, D. Jackson, and M. Zhao, "The provable security of Ed25519: Theory and practice," in *2021 IEEE Symposium on Security and Privacy*, (San Francisco, CA, USA), pp. 1659–1676, IEEE Computer Society Press, May 24–27, 2021.

K. Chalkias, F. Garillot, and V. Nikolaenko, "Taming the many EdDSAs." Cryptology ePrint Archive, Report 2020/1244, 2020.

`https://eprint.iacr.org/2020/1244`.

G. Fuchsbauer, A. Plouviez, and Y. Seurin, "Blind schnorr signatures and signed ElGamal encryption in the algebraic group model," in *Advances in Cryptology – EUROCRYPT 2020, Part II* (A. Canteaut and Y. Ishai, eds.), vol. 12106 of *Lecture Notes in Computer Science*, (Zagreb, Croatia), pp. 63–95, Springer, Heidelberg, Germany, May 10–14, 2020.

D. J. Bernstein, N. Duif, T. Lange, P. Schwabe, and B.-Y. Yang, "High-speed high-security signatures," in *Cryptographic Hardware and Embedded Systems – CHES 2011* (B. Preneel and T. Takagi, eds.), vol. 6917 of *Lecture Notes in Computer Science*, (Nara, Japan), pp. 124–142, Springer, Heidelberg, Germany, Sept. 28 – Oct. 1, 2011.

D. J. Bernstein, S. Josefsson, T. Lange, P. Schwabe, and B.-Y. Yang, "EdDSA for more curves." Cryptology ePrint Archive, Report 2015/677, 2015.

`https://eprint.iacr.org/2015/677`.

S. Josefsson and I. Liusvaara, "Edwards-Curve Digital Signature Algorithm (EdDSA)," Request for Comments RFC 8032, Internet Engineering Task Force, Jan. 2017.

Num Pages: 60.

D. Moody, "Digital Signature Standard (DSS)," Tech. Rep. NIST FIPS 186-5, National Institute of Standards and Technology, Gaithersburg, MD, 2023.

J. T. Schwartz, "Fast Probabilistic Algorithms for Verification of Polynomial Identities," *Journal of the ACM*, vol. 27, pp. 701–717, Oct. 1980.


D. Hofheinz, T. Jager, and E. Kiltz, "Short signatures from weaker assumptions," in *Advances in Cryptology – ASIACRYPT 2011* (D. H. Lee and X. Wang, eds.), vol. 7073 of *Lecture Notes in Computer Science*, (Seoul, South Korea), pp. 647–666, Springer, Heidelberg, Germany, Dec. 4–8, 2011.